

$$\begin{aligned}
&= X \cdot X \cdot Y + X \cdot Y \cdot \bar{Z} + \bar{X} \cdot \bar{X} \cdot Z + \bar{X} \cdot \bar{Y} \cdot Z \\
&= X \cdot Y(X + \bar{Z}) + (\bar{X} + \bar{Y})\bar{X} \cdot Z \\
&= X \cdot Y(\overline{\bar{X} \cdot Z}) + (\bar{X} \cdot \bar{Y})\bar{X} \cdot Z \\
&= (X \cdot Y) \oplus (\bar{X} \cdot Z) \quad [2.74]
\end{aligned}$$

## 2.10. Exercises

EXERCISE 2.1.– Function of  $n$  variables.

How many different logic functions can we implement using  $n$  input variables?

EXERCISE 2.2.– Analysis of a logic circuit.

Construct the truth table for the circuit shown in Figure 2.62. Using a minimum number of NAND gates, propose a circuit to implement  $F$ .

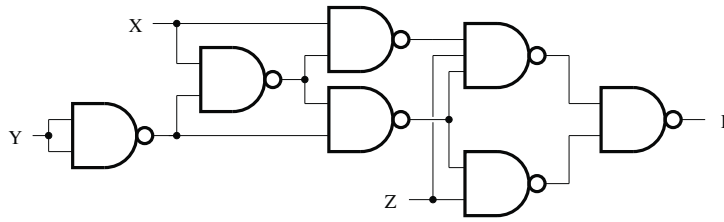


Figure 2.62. Logic circuit

EXERCISE 2.3.– Simplification of logic functions.

Simplify the following logic expressions:

- $X + \bar{Y} + \bar{X} \cdot Y + (X + \bar{Y}) \cdot \bar{X} \cdot Y$
- $X \cdot Y \cdot Z + X \cdot \bar{Y} \cdot Z + \bar{X} \cdot Y \cdot Z + \bar{X} \cdot Y \cdot \bar{Z}$
- $\overline{\bar{X} \cdot Y \cdot \bar{X} \cdot Z}$
- $(X + \bar{Y})(\bar{X} + Z)(Y + \bar{Z})$
- $(W + X + Y \cdot Z)(\bar{W} + X)(\bar{X} + Y)$
- $W \cdot X \cdot \bar{Z} + \bar{X} \cdot Y \cdot Z + W \cdot X \cdot \bar{Y} + X \cdot Y \cdot Z + \bar{W} \cdot Y \cdot Z$
- $\bar{W} \cdot X \cdot Z + W \cdot Z + X \cdot Y \cdot \bar{Z} + \bar{W} \cdot X \cdot Y$
- $(X + Y + Z)(\bar{X} + Y + Z)(\bar{X} + Y + \bar{Z})$

## EXERCISE 2.4.– Equivalent logic functions.

Verify the following equalities:

$$\text{a) } \overline{A} \cdot B \cdot \overline{C} + A \cdot C + \overline{B} \cdot C = C \oplus (\overline{A} \cdot B)$$

$$\text{b) } A \oplus B \oplus (A + B) + \overline{A} \cdot \overline{B} = A \odot B$$

$$\text{c) } (A + B) \odot (\overline{A} + C) + A = A + B$$

$$\text{d) } (A \odot B) \oplus (A \cdot \overline{B}) + A \cdot \overline{B} = A + \overline{B}$$

$$\text{e) } \overline{A} \cdot \overline{B} \cdot \overline{C} + \overline{A} \cdot \overline{C} \cdot \overline{D} + \overline{A} \cdot B \cdot C \cdot D + A \cdot B \cdot \overline{C} \cdot D + A \cdot C \cdot \overline{D} + A \cdot \overline{B} \cdot C = A \cdot C \oplus \overline{A} \cdot \overline{C} \oplus B \cdot D$$

$$\text{f) } A \cdot B \cdot \overline{D} + \overline{B} \cdot C \cdot D + A \cdot B \cdot \overline{C} + B \cdot C \cdot D + \overline{A} \cdot C \cdot D = A \cdot B + C \cdot D$$

## EXERCISE 2.5.– Simplification of the functions using the Karnaugh map method.

Using the Karnaugh map method, simplify each of the following logic functions:

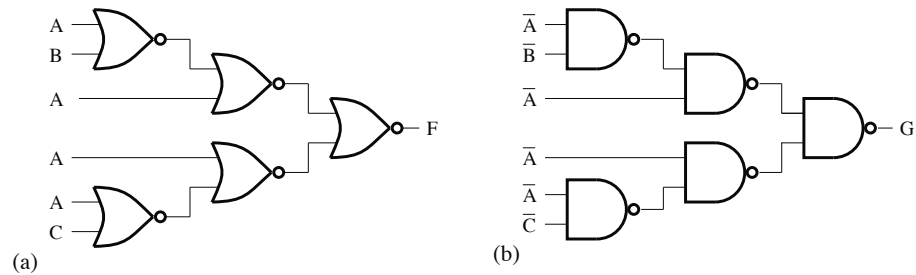
$$E(A, B, C, D) = \sum mm(0, 2, 3, 4, 5, 8, 11, 12, 13, 14, 15) \quad [2.75]$$

$$F(A, B, C, D) = \sum m(0, 1, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14, 15) \quad [2.76]$$

$$G(A, B, C, D) = \sum m(0, 2, 4, 5, 10, 12, 15) + \sum x(8, 14) \quad [2.77]$$

$$H(A, B, C, D) = \sum mm(1, 3, 6, 8, 11, 14) + \sum x(2, 5, 12, 13, 15) \quad [2.78]$$

## EXERCISE 2.6.– Analysis of the circuit with NAND/NOR gates.

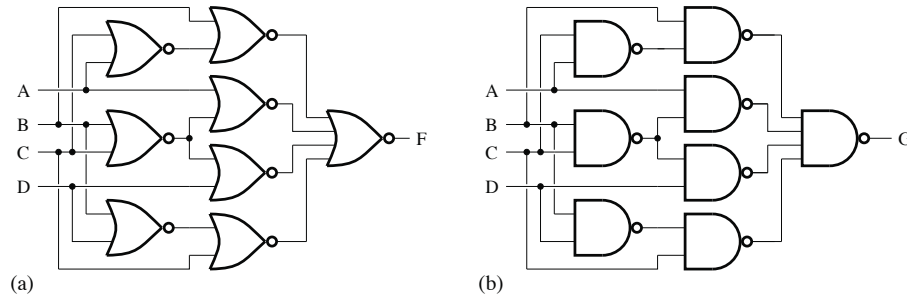


**Figure 2.63.** a) Circuit with NAND gates and b) circuit with NOR gates

Determine the logic function  $F$  implemented by the circuit shown in Figure 2.63(a). Determine the logic function  $G$  implemented by the circuit shown in Figure 2.63(b). Verify that  $F = \overline{G}$ .

EXERCISE 2.7.– Canonical forms of a logic function.

Let us consider the logic functions represented in Figure 2.64.



**Figure 2.64.** a) Circuit with NOR gates and b) circuit with NAND gates

Determine the logic expressions for the outputs  $F$  and  $G$ .

Express the logic function  $F$  as a sum of products.

Give the product-of-sums form of  $G$ .

EXERCISE 2.8.– Simplification of logic circuits.

Simplify each of the logic circuits shown in Figure 2.65.

EXERCISE 2.9.– Implementation of the function  $H$ .

Let us consider the three-variable logic function defined as follows:

$$H(A, B, C) = A \cdot \overline{B} + \overline{A} \cdot B + B \cdot \overline{C} \quad [2.79]$$

Assuming the input variables are  $A$ ,  $B$  and  $C$ , propose a logic circuit using only 2-input NAND gates to implement the function  $H$ .

EXERCISE 2.10.– Implementation of the logic function  $Y$ .

Using the minimum number of logic gates (NOT, AND, OR) to realize the following logic function:

$$Y = (A \cdot B) \oplus (\overline{B} \cdot \overline{C}) \quad [2.80]$$

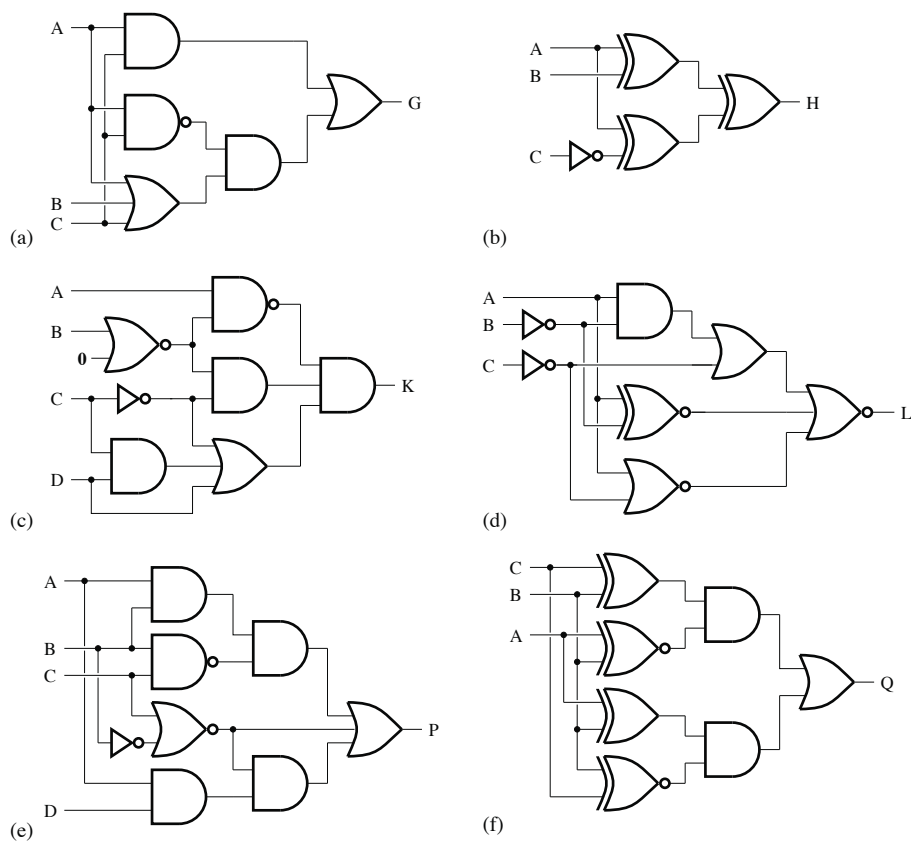


Figure 2.65. Logic circuits

EXERCISE 2.11.– Implementation of circuits with NAND/NOR gates.

Let us consider the following logic functions:

$$P(A, B, C, D) = A \cdot B + \bar{A} \cdot \bar{C} \cdot \bar{D} \quad [2.81]$$

$$\text{and } Q(A, B, C, D) = (\bar{A} + \bar{B} + \bar{C})(A + D) \quad [2.82]$$

Assuming that the input variables  $A$ ,  $B$ ,  $C$ , and  $D$ , as well as their complements, are available, propose a logic circuit based on 2-input NAND gates and a logic circuit based on 2-input NOR gates for each of these functions.